De-noising 'Initial Condition Modulation' Wideband Chaotic Communication Systems with Linear & Wavelet Filters

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Abstract

In this paper de-noising techniques are investigated in connection with secure wideband chaotic communication systems. An alternate version of the recently proposed Ueda chaotic communication system based on the initial condition modulation (ICM) of the chaotic carrier by the binary message to be transmitted is proposed and evaluated in the presence of noise, demonstrating a significant improvement. It is then shown that the running average finite impulse response (FIR) filter, and the hard-threshold filtering techniques in Haar and Daubechies wavelet domain can be used to significantly improve the performance of the proposed chaotic communication system.

1. Introduction

The motivation of this paper is to investigate the performance of filtering techniques when applied to the wideband chaotic communication system. Noise removal from chaotic time series has been attempted by a number of researchers [1, 2, 3, 4, 5, 6, 7], among others, and is still an active area of research. Filtering methods include linear filters [2] and different wavelet techniques [1, 3, 4, 5], among other. A potential application of chaotic filtering techniques lies in chaotic communication systems [6, 7]. Since the onset of chaotic synchronization [8] a number of chaotic communication systems based on chaotic synchronization have been proposed, for example [9, 10, 11]. This paper applies running average FIR filter and hard-threshold filtering (de-noising) technique in Haar and Daubechies wavelet domain [12], to the herein proposed Ueda chaotic communication system based on the "initial condition modulation" (ICM) of the chaotic carrier by the binary message to be transmitted [9]. It will be shown in terms of the bit error rate that the filtering techniques presented significantly improve the performance of the wideband communication system proposed.

Time series plots and phase space diagrams [13] are also used to pictorially represent the effect of the filtering.

2. Haar wavelet transform

The wavelet transform is based on the approximation of a time domain function *'f'* in the time-frequency domain. The Haar wavelet transform uses two consecutive time domain values of function *'f'* to represent them by one wider step and one wavelet [12] in the new domain, that is, time-frequency domain. The wider step measures the average while the wavelet measures the difference between the two consecutive values and divides this difference by two.

The forward Haar wavelet transform is given by equations 2.1-a and 2.1-b, and the inverse Haar wavelet transform by equations 2.2-a and 2.2-b [12].

$$a_k^{l+1} = \frac{a_{2k-1}^l + a_{2k}^l}{2}, \qquad c_k^{l+1} = \frac{a_{2k-1}^l - a_{2k}^l}{2} \tag{2.1-a,b}$$

$$a_{2k-1}^l = a_k^{l+1} + c_k^{l+1}, \qquad a_{2k}^l = a_k^{l+1} - c_k^{l+1} \tag{2.2-a,b}$$

In equations 2.1 and 2.2, '*a*' designates the average coefficients and '*c*' the wavelet coefficients. The average and wavelet coefficients are organised into the *Average* and *Wavelet* matrices of the form shown below.

$$Average = \begin{bmatrix} a_1^1 & a_2^1 & a_3^1 & a_4^1 \\ a_1^2 & a_2^2 & 0 & 0 \\ a_1^3 & 0 & 0 & 0 \end{bmatrix} \qquad Wavelet = \begin{bmatrix} c_1^1 & c_2^1 & c_3^1 & c_4^1 \\ c_1^2 & c_2^2 & 0 & 0 \\ c_1^3 & 0 & 0 & 0 \end{bmatrix}$$

3. Daubechies wavelet transform

Unlike Haar wavelet transform which exhibits jump discontinuities in the signal transformation, Daubechies wavelet transform is a smoother approximation based on Daubechies basis function [12]. The forward Daubechies wavelet transform is given by equations 3.1-a and 3.1-b, and the inverse by equations 3.2-a and 3.2-b [12] (note that in [12] printing errors have caused erroneous equations, here they have been revised).

$$a_{k+1}^{l+1} = \frac{1}{2} \cdot (h_0 \cdot a_{2k+1}^{l} + h_1 \cdot a_{2k+2}^{l} + h_2 \cdot a_{2k+3}^{l} + h_3 \cdot a_{2k+4}^{l}) \tag{3.1-a}$$

$$c_{k+1}^{l+1} = \frac{1}{2} \cdot (h_3 \cdot a_{2k+1}^{l} - h_2 \cdot a_{2k+2}^{l} + h_1 \cdot a_{2k+3}^{l} - h_0 \cdot a_{2k+4}^{l}) \tag{3.1-b}$$

$$a_{k+1}^{l} = h_2 \cdot a_{k}^{l+1} + h_1 \cdot c_{k}^{l+1} + h_0 \cdot a_{k+1}^{l+1} + h_3 \cdot c_{k+1}^{l+1} \tag{3.2-a}$$

$$a_{k+2}^{l} = h_3 \cdot a_{k}^{l+1} - h_0 \cdot c_{k}^{l+1} + h_1 \cdot a_{k+1}^{l+1} - h_2 \cdot c_{k+1}^{l+1} \tag{3.2-b}$$

4. Hard-thresholding in the wavelet domain

The underlying idea of the hard-threshold filtering technique is based on looking at the average power of the wavelet scales, that is, at each row of the *Wavelet* matrix. Hard-thresholding in the wavelet domain involves deletion of certain wavelet scales where noise exists, but signal does not [12]. In other words, provided that the pure signal does not contain significant average power in certain row of the *Wavelet* matrix, but when mixed with noise the average power increases in that particular row, then this row can be set to zero in its entirety.

5. Application to communications

5.1. Master – slave communications

Since Pecora and Carroll showed that two chaotic systems can synchronize, and suggested the application of chaos synchronization in communications [8], a number of chaotic communication systems based on chaotic synchronization emerged [9, 10, 11], among other. The Pecora-Carroll synchronization scheme can be viewed as a master-slave synchronization system. The master system provides at least one of its outputs to the slave system. The slave system uses the given master output (driving signal), to synchronize itself to the master system (or not), regardless of its initial conditions. The master-slave system can also be viewed as the transmitter-receiver communication system as is described in what follows.

5.2. Initial condition modulation scheme

In [9] the authors proposed a digital communication system, based on the initial condition modulation of the chaotic carrier by the binary message to be transmitted. The demodulation process at the receiver is based on the synchronization of one of the master-slave signals. Due to the smooth nature of the transmitted signal at the bit transitions, it was argued in [9] that the Ueda chaotic communication system is the most secure out of the systems examined. In case of the Ueda chaotic communication system, when the master x signal drives, demodulation at the receiver is based on the chaotic synchronization properties of the master-slave y signals, governed by equation 5.1 [9].

$$y(t_o) - \hat{y}(t_o) = A - k\int_{t=0}^{t=t_o} (y(t) - \hat{y}(t))dt + 2B\sin(\phi)\sin(t_o + \Omega) \tag{5.1}$$

where: $\phi = \frac{z - \hat{z}}{2} = \frac{z(0) - \hat{z}(0)}{2}$, $\Omega = \phi + \hat{z}(0) + \pi/2$.

In equation 5.1 '^' above the y signal represents the slave y signal. In [9] it has been shown that equation 5.1 settles to steady state behaviour, governed by its third term, as time tends to infinity. As the third term of equation 5.1 is governed by the initial conditions of the master-slave z signals, $z(0)$ and $\hat{z}(0)$, respectively, it has been shown that equation 5.1 can be used to demodulate the received binary message, given that the binary message is represented by the difference of the master-slave z initial conditions [9]. Furthermore it has also been shown that the separation of the binary symbols in their symbol space is largest when the difference among the master-slave z initial conditions is equal to $\pm 2n\pi$ and $\pm n\pi$, depending on whether binary 0 or binary 1 are transmitted, respectively.

In order to implement equation 5.1 at the receiver (slave) side, it is required to transmit both the driving transmitter (master) signal x, and the master signal y.

5.3. Proposed alternate Ueda chaotic communication system

It is now shown how the Ueda chaotic communication system of [9] can be modified to simplify its structure, while at the same time improving the noise performance. In [9] demodulation of the received signal is achieved by observing the synchronization error of the master-slave y signals. If this error tends to zero, it is concluded by the receiver that bit 0 was sent. It is concluded that bit 1 was sent if the steady state error is sinusoidal, that is, not zero. Here it is graphically shown (Fig. 2) that it is sufficient to only observe the behaviour of the slave signal y in order to successfully discriminate among the binary symbols 0 and 1. In this case it is required to only transmit the driving transmitter (master) signal x, thus reducing the required bandwidth. Such a communication system is presented in Fig. 1.

In Fig. 1 the message m is varied among 2π and π, depending on whether bit 0 or bit 1 is to be transmitted, respectively. In order to ensure continuity of the smooth nature of the transmitted signal x, as well as to avoid periodicity of chaotic sequences representing bit 0 and bit 1, the initial conditions of x (y), for every new bit transmitted, are chosen as the final values of the chaotic carrier (signal) of the preceding bit [9]. Note that in Fig. 1, n represents the additive white Gaussian noise (AWGN).

The transmitted signal x and the squared slave signal $\hat{y}$ are shown in Figs. 5a and 2, respectively, when m = [2π, 2π, π, 2π, π, π, 2π, π, 2π, π], or in binary terms: *message* = [0 0 1 0 1 1 0 1 0 1]. From Fig. 5a observe the smooth nature of the transmitted signal x. In Fig. 2 the dominant peaks represent bits 1.

In order to compare the performance, under AWGN, of the Ueda chaotic communication system of Fig. 1 to the Ueda chaotic communication system of [9] an empirical bit error rate (BER) curve has been produced and shown in Fig. 7 by the open circles. As in the evaluation of most binary modulation techniques, clock synchronization among the transmitter and receiver has been assumed [9]. The spreading factor, that is, the number of chaotic points representing each bit has been chosen to be 400 [9]. For comparison the BER curves for the parameter modulation chaotic communication system of [11], and for the BPSK communication system, have been produced and denoted by crosses and solid line, respectively.

5.4. Running average FIR filtering

In [2] it has been reported that the linear filters can be used to filter non-linear systems. It is now shown that the noise performance of the system proposed in sub-section 5.3 can be further improved by using the running average finite impulse response (FIR) filter [14] to filter the received signal $x_r(t)$.

The noisy Ueda chaotic signal x at Eb/No of 25 dB is shown in Fig. 5b, and the corresponding filtered signal is shown in Fig. 5c. The Ueda strange attractor is shown in Fig. 6a, followed by the noisy and filtered Ueda strange attractor at Eb/No of 25 dB in Figs. 6b and 6c, respectively.

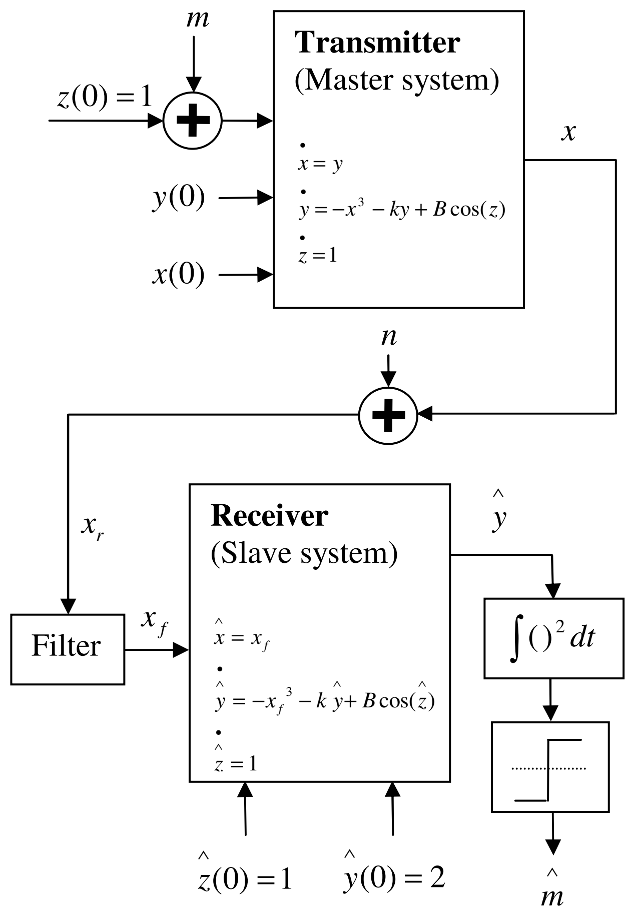

Fig. 1 The Ueda chaotic communication system, based on the initial condition modulation of [9], but with only x transmitted

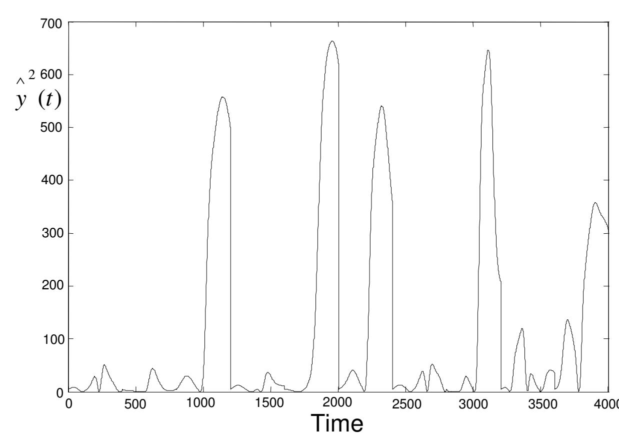

Fig. 2 Slave signal $\hat{y}$ squared

From these a clear improvement in the filtered signal can be noticed as compared to the noisy one. The effect of this filtering technique on the communication system of Fig. 1 is demonstrated in Fig. 7 by the bit error rate (BER) curve marked by the pentagram stars. Comparing this BER curve to the BER curve of the system of Fig. 1 without filter (marked by the open circles), an improvement of 3-4 dB can be observed.

5.5. Filtering in the Haar wavelet domain

Fig. 3 shows the Ueda chaotic signal x in Haar wavelet domain, for three different noise levels, and for the case when there is no noise. From Fig. 3 it can be seen that the first five rows of the *Wavelet* matrix can be hard-threshold to zero at bit energy to noise power spectral density ratio (Eb/No) of 20 dB and 15 dB and the first four rows at Eb/No of 25 dB.

The Haar filtered Ueda chaotic signal x_f is shown in Fig. 5d.

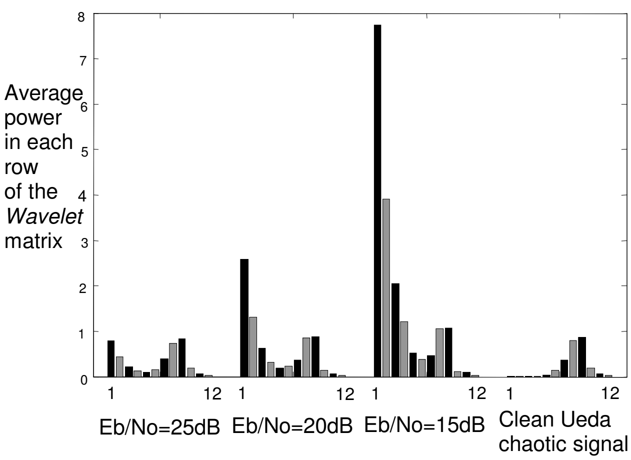

Fig. 3 Average power of the coefficients in each row of the Haar *Wavelet* matrix for the noise polluted signal, where Eb/No is 25 dB, 20 dB and 15 dB, and for the clean Ueda chaotic signal x

The 'Haar' filtered Ueda strange attractor is presented in Fig. 6d. The prominent edge effects can be observed in Fig. 6d. These edge effects can be explained in the following manner. Each row of the *Wavelet* matrix is a rough approximation of the row above it, for example, second row is a rough approximation of the first row. Hard-threshold to zero of the first four rows of the *Wavelet* matrix allows lower rows to estimate the upper rows.

In Fig. 7 the BER curve of the system of Fig. 1, when filtered in the Haar wavelet domain, is represented by the asterisks. It demonstrates an improvement of about 3-4 dB as compared to the non-filtered one (open circles).

5.6. Filtering in the Daubechies wavelet domain

Fig. 4 shows the Ueda chaotic signal x in Daubechies wavelet domain, for three different noise levels, and for the case when there is no noise. From Fig. 4 it can be seen that for the Ueda chaotic signal x the first five rows of the *Wavelet* matrix can be hard-threshold to zero at Eb/No of 20 dB and 15 dB, and the first four rows at Eb/No of 25 dB.

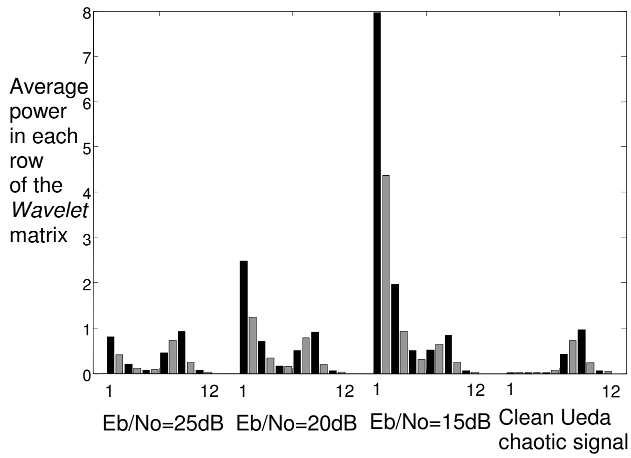

Fig. 4 Average power of the coefficients in each row of the Daubechies *Wavelet* matrix for the noise polluted signal, where Eb/No is 25 dB, 20 dB and 15 dB, and for the clean Ueda chaotic signal x

The Daubechies filtered time series and the corresponding chaotic attractor are shown in Figs. 5e and 6e, respectively.

As with Haar, using the Daubechies filtering technique to filter $x_r(t)$, and thus produce $x_f(t)$, improves the BER curve of the system of Fig. 1 by 3-4 dB, as is demonstrated in Fig. 7 by the curve marked by the solid squares.

5.7. Results and Discussions

The bit error rate curves demonstrating the noise performance of the aforementioned communication systems are displayed in Fig. 7. The Ueda initial condition modulation wideband chaotic communication system, proposed in section 5.3 (open circles), exhibits an improvement in the BER curve of approximately 5 dB as compared to the Ueda chaotic communication system of [9] (full circles). The three filtering techniques presented above all improve the BER curve of the non-filtered system of Fig. 1 by 3-4 dB with the running average FIR filter exhibiting marginally better performance than the Haar and Daubechies filtering techniques.

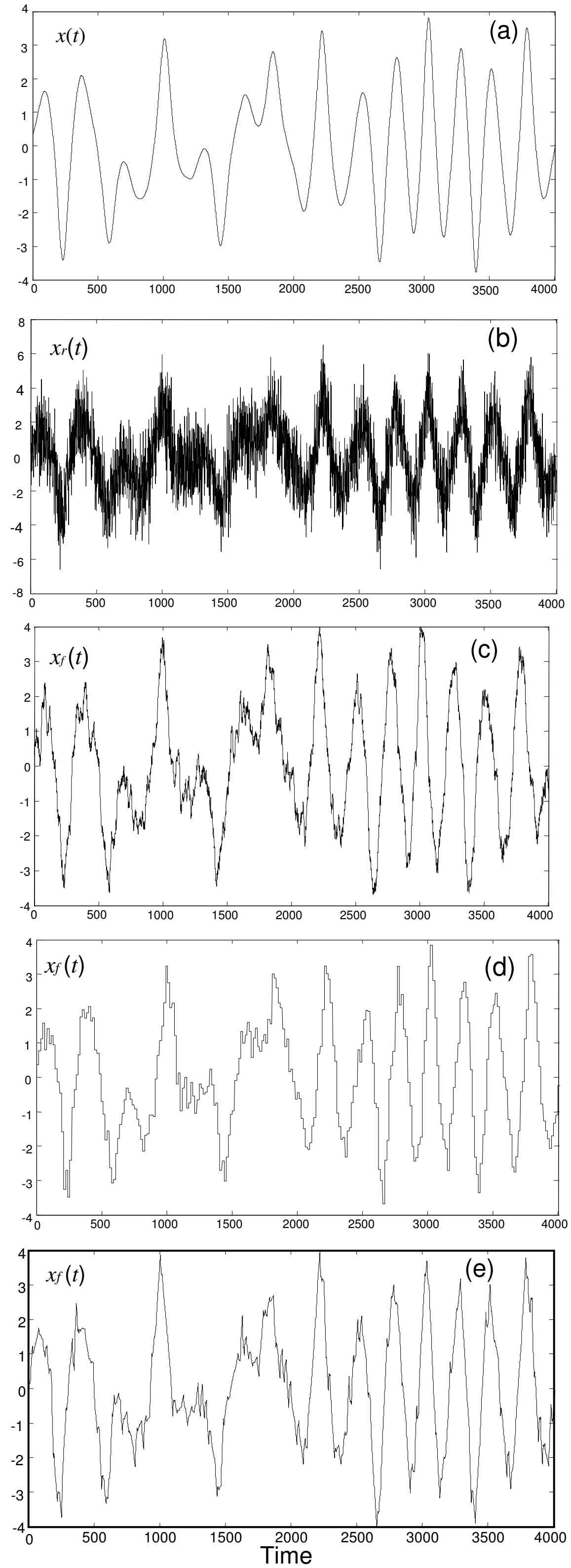

Fig. 5 (a) Transmitted signal x, (b) Received signal x_r at Eb/No = 25 dB, (c) FIR filtered signal x_f, (d) Haar filtered signal x_f, (e) Daubechies filtered signal x_f

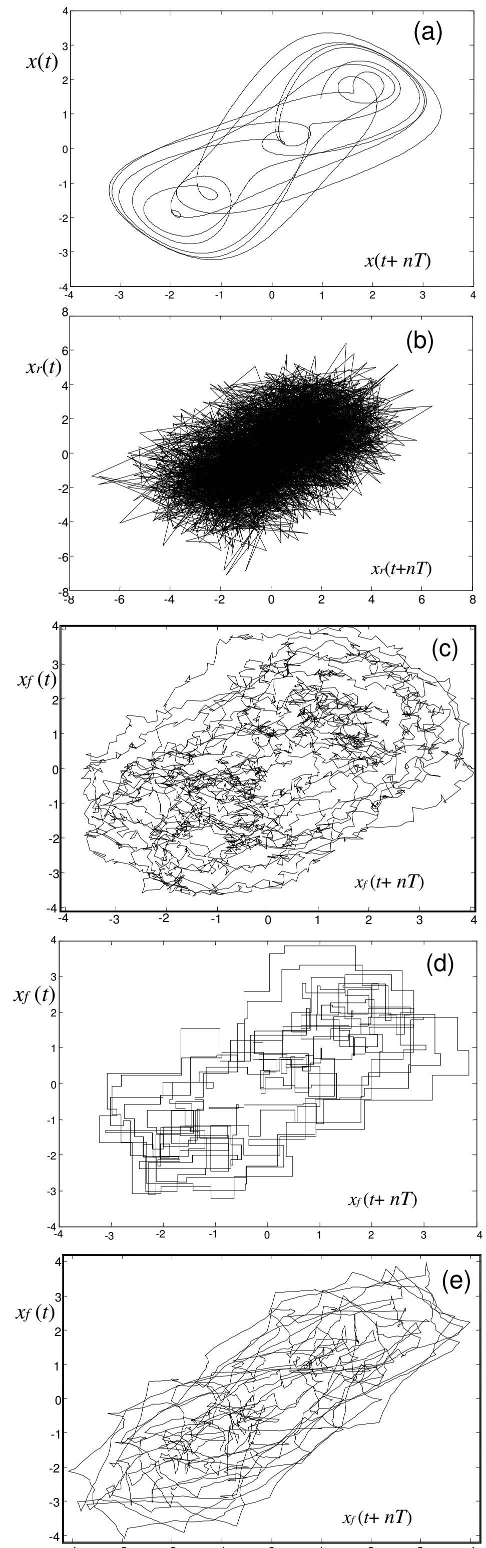

Fig. 6 Ueda strange attractor: (a) Clean, (b) at Eb/No = 25 dB, (c) FIR filtered, (d) Haar filtered, (e) Daubechies filtered

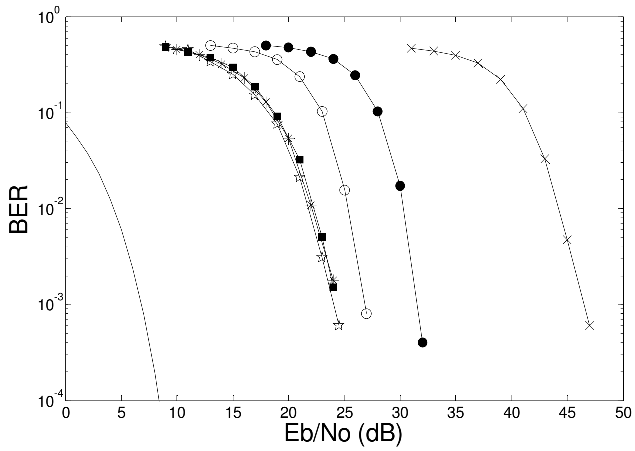

Fig. 7 The BER curves: (a) the solid line is for the theoretical BPSK, (b) the pentagram stars are for the running average FIR filtered system of Fig. 1, (c) the asterisks are for the Haar filtered system of Fig. 1, (d) the solid squares are for the Daubechies filtered system of Fig. 1, (e) the open circles are for the non filtered system of Fig. 1, (f) the solid circles are for the Ueda initial condition modulation system introduced in [9], (g) the crosses are for the Lorenz chaotic parameter modulation system introduced in [11].

From Fig. 7 it is noted that for the Ueda chaotic communication system of Fig. 1 (open circles) it requires 18-19 dB less energy per bit to achieve the same probability of error as compared to the chaotic parameter modulation system of [11] (crosses). Filtering techniques improve the performance even further. The system of [11] could not be efficiently filtered, using aforementioned filtering techniques, due to the very low levels of noise in its operating region, with an attempt at filtering worsening the performance.

6. Conclusions

It has been shown that the herein proposed alternative of the secure Ueda chaotic communication system, based on the recently proposed technique of initial condition modulation, exhibits a significant improvement in terms of the bit error rate. The running average FIR filter and the hard-threshold wavelet de-noising techniques, in Haar and Daubechies wavelet domain, have been described and applied to the proposed secure communication system, demonstrating another significant improvement in the bit error rate curve. In [2] it has been mentioned that the linear filters can be used to filter non-linear systems. Here this has been demonstrated by applying the running average FIR filter within a wide band chaotic communication system. It has also been shown that smooth chaotic time-series of the Ueda chaotic communication system can be successfully filtered in wavelet domain, using hard-threshold to zero filtering technique. Despite the fact that filtering in wavelet domain introduces edge effects the dynamics of the strange attractor seem to be well preserved. Filtering in Daubechies wavelet domain exhibits smoother edge effects on strange attractor than filtering in Haar wavelet domain. Our future work will involve investigation of chaotic maps for wideband communication systems.

7. References

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